# *PAST EXAM QUESTIONS*

This section contains all the Subject CT1 exam questions from the period 2008 to 2017 that are related to the topics covered in this booklet.

Solutions are given later in this booklet. These give enough information for you to check your answer, including working, and also show you what an adequate examination answer should look like. Further information may be available in the Examiners' Report, ASET or Course Notes. (ASET can be ordered from ActEd.)

We first provide you with a **cross reference grid** that indicates the main subject areas of each exam question. You can use this, if you wish, to select the questions that relate just to those aspects of the topic that you may be particularly interested in reviewing.

Alternatively you can choose to ignore the grid, and instead attempt each question without having any clues as to its content.

***Cross reference grid***

| | | | Basic interest | | | | Variable force of interest | | | Annuities | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Question*** | Tick when attempted | Cashflow models | Accumulation/ discount | Converting | Treasury bill | Nominal interest/discount | Accumulation/ discount | General A(t) / v(t) expression | Payment streams | Level annuities | Increasing annuities |
| 1 | | | | | | ✓ | ✓ | | ✓ | | |
| 2 | | | | | | | ✓ | | | | |
| 3 | | ✓ | | | | | | | | | |
| 4 | | | | ✓ | ✓ | | | | | | |
| 5 | | | | | | | ✓ | | | | |
| 6 | | | | | | | ✓ | ✓ | ✓ | | |
| 7 | | | | | | ✓ | ✓ | ✓ | ✓ | | |
| 8 | | | | ✓ | | ✓ | ✓ | | | | |
| 9 | | | | ✓ | ✓ | | | | | | |
| 10 | | | | | | | | | | ✓ | |
| 11 | | | | | | | ✓ | | | | |
| 12 | | | | | | | ✓ | | ✓ | | |
| 13 | | | | ✓ | ✓ | | | | | | |
| 14 | | | | ✓ | | ✓ | | | | | |
| 15 | | | | ✓ | | | ✓ | ✓ | ✓ | | |
| 16 | | | | | | | ✓ | | ✓ | | |
| 17 | | | | ✓ | | ✓ | | | | | |
| 18 | | | | | | | ✓ | | ✓ | | |
| 19 | | | | ✓ | | ✓ | | | | | |
| 20 | | | | ✓ | | | ✓ | | (✓) | | |
| 21 | | | ✓ | (✓) | | | | | | | |
| 22 | | | | | | | | | | ✓ | ✓ |
| 23 | | | | ✓ | | | ✓ | | ✓ | | |
| 24 | | | ✓ | | | | | | | | |
| 25 | | | | | ✓ | | | | | | |
| 26 | | | ✓ | | | ✓ | | | | ✓ | |
| 27 | | | | | | | ✓ | ✓ | ✓ | | |
| 28 | | | ✓ | | ✓ | | | | | | |
| 29 | | | | ✓ | | ✓ | | | | | |

| | | | Basic interest | | | | Variable force of interest | | | Annuities | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Question*** | Tick when attempted | Cashflow models | Accumulation/ discount | Converting | Treasury bill | Nominal interest/discount | Accumulation/ discount | General A(t) / v(t) expression | Payment streams | Level annuities | Increasing annuities |
| 30 | | | | | | | ✓ | | ✓ | | |
| 31 | | | | | | ✓ | ✓ | | | | |
| 32 | | | | ✓ | | ✓ | | | | | |
| 33 | | | | | | ✓ | | | | ✓ | |
| 34 | | ✓ | | | | | | | | | |
| 35 | | | | | ✓ | | | | | | |
| 36 | | | | | | | ✓ | | ✓ | | |
| 37 | | | | ✓ | | ✓ | | | | | |
| 38 | | | ✓ | ✓ | | | | | | | |
| 39 | | | | | ✓ | | | | | | |
| 40 | | | | | | | | | | ✓ | |
| 41 | | | | | | ✓ | ✓ | | ✓ | | |

**1** ***Subject CT1 April 2008 Question 9***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t)=\begin{cases}0.06 & 0\le t\le 4\\ 0.10-0.01t & 4<t\le 7\\ 0.01t-0.04 & 7<t\end{cases}$$

(i) Calculate the value at time $t=5$ of £1,000 due for payment at time $t=10$. [5]

(ii) Calculate the constant rate of interest per annum convertible monthly which leads to the same result as in (i) being obtained. [2]

(iii) Calculate the accumulated amount at time $t=12$ of a payment stream, paid continuously from time $t=0$ to $t=4$, under which the rate of payment at time $t$ is $\rho(t)=100e^{0.02t}$. [6]

[Total 13]

**2** ***Subject CT1 September 2008 Question 7***

The force of interest, $\delta(t)$, is a function of time and at any time $t$ (measured in years) is given by:

$$\delta(t)=\begin{cases}0.05+0.02t & \text{for } 0\le t\le 5\\ 0.15 & \text{for } t>5\end{cases}$$

(i) Calculate the present value of £1,000 due at the end of 12 years. [5]

(ii) Calculate the annual effective rate of discount implied by the transaction in (i). [2]

[Total 7]

**3** ***April 2009 Question 2***

Describe the characteristics of:

(a) an interest-only loan (or mortgage); and

(b) a repayment loan (or mortgage). [4]

**4** ***Subject CT1 September 2009 Question 1***

A 182-day government bill, redeemable at £100, was purchased for £96 at the time of issue and was later sold to another investor for £97.89. The rate of return received by the initial purchaser was 5% per annum effective.

(a) Calculate the length of time in days for which the initial purchaser held the bill.

(b) Calculate the annual simple rate of return achieved by the second investor. [4]

**5** ***Subject CT1 September 2009 Question 5***

The force of interest $\delta(t)$ at time $t$ is $a + bt^2$ where $a$ and $b$ are constants. An amount of £100 invested at time $t = 0$ accumulates to £130 at time $t = 5$ and £200 at time $t = 10$ .

(i) Calculate the values of $a$ and $b$ . [6]

(ii) Calculate the constant rate of interest per annum convertible monthly that would give rise to the same accumulation from time $t = 0$ to time $t = 5$ . [2]

(iii) Calculate the constant force of interest that would give rise to the same accumulation from time $t = 5$ to time $t = 10$ . [2]

[Total 10]

 

**6** ***Subject CT1 April 2010 Question 11***

The force of interest $\delta(t)$ is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 + 0.02t & 0 \le t < 5 \\ 0.05 & 5 \le t \end{cases}$$

(i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time $t$. [5]

(ii) (a) Calculate the present value of £1,000 due at the end of 17 years.

(b) Calculate the rate of interest per annum convertible monthly implied by the transaction in part (ii)(a). [4]

A continuous payment stream is received at a rate of $10e^{0.01t}$ units per annum between $t = 6$ and $t = 10$.

(iii) Calculate the present value of the payment stream. [4]
[Total 13]

**7** ***Subject CT1 September 2010 Question 8***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \le t \le 20 \\ 0.05 & t > 20 \end{cases}$$

(i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time $t$. [5]

(ii) (a) Calculate the present value of £100 due at the end of 25 years.

(b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a). [4]

(iii) A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between $t = 20$ and $t = 25$. Calculate the accumulated value of the payment stream at time $t = 25$. [4]
[Total 13]

**8** ***Subject CT1 April 2011 Question 1***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for} \quad 0 < t \le 5 \\ 0.01 + 0.03t & \text{for} \quad 5 < t \end{cases}$$

(i) Calculate the amount to which £1,000 will have accumulated at $t = 7$ if it is invested at $t = 3$. [4]

(ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (i) being obtained. [3]

[Total 7]

**9** ***Subject CT1 September 2011 Question 1***

A 91-day treasury bill is issued by the government at a simple rate of discount of 8% per annum.

Calculate the annual effective rate of return obtained by an investor who purchases the bill at issue. [3]

**10** ***Subject CT1 September 2011 Question 3***

An individual intends to retire on his 65th birthday in exactly four years' time. The government will pay a pension to the individual from age 68 of £5,000 per annum monthly in advance. The individual would like to purchase an annuity certain so that his income, including the government pension, is £8,000 per annum paid monthly in advance from age 65 until his 78th birthday. He is to purchase the annuity by a series of payments made over four years quarterly in advance starting immediately.

Calculate the quarterly payments the individual has to make if the present value of these payments is equal to the present value of the annuity he wishes to purchase at a rate of interest of 5% per annum effective. Mortality should be ignored. [6]

**11** ***Subject CT1 September 2011 Question 6***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is $a+bt$ where $a$ and $b$ are constants. An amount of £45 invested at time $t=0$ accumulates to £55 at time $t=5$ and £120 at time $t=10$.

(i) Calculate the values of $a$ and $b$. [5]

(ii) Calculate the constant force of interest per annum that would give rise to the same accumulation from time $t=0$ to time $t=10$. [2]
[Total 7]

**12** ***Subject CT1 April 2012 Question 8***

The force of interest, $\delta(t)$, at time $t$ is given by:

$$\delta(t)=\begin{cases} 0.04+0.003t^2 & \text{for} \quad 0<t\le 5 \\ 0.01+0.03t & \text{for} \quad 5<t\le 8 \\ 0.02 & \text{for} \quad t>8 \end{cases}$$

(i) Calculate the present value (at time $t=0$) of an investment of £1,000 due at time $t=10$. [4]

(ii) Calculate the constant rate of discount per annum convertible quarterly, which would lead to the same present value as that in part (i) being obtained. [2]

(iii) Calculate the present value (at time $t=0$) of a continuous payment stream payable at the rate of $100e^{0.01t}$ from time $t=10$ to $t=18$. [4]
[Total 10]

**13** ***Subject CT1 September 2012 Question 1***

An investor is considering two investments. One is a 91-day deposit which pays a rate of interest of 4% per annum effective. The second is a treasury bill.

Calculate the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]

**14** ***Subject CT1 September 2012 Question 2***

The nominal rate of discount per annum convertible quarterly is 8%.

(i) Calculate the equivalent force of interest. [1]

(ii) Calculate the equivalent effective rate of interest per annum. [1]

(iii) Calculate the equivalent nominal rate of discount per annum convertible monthly. [2]

[Total 4]

**15** ***Subject CT1 September 2012 Question 8***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.03 + 0.01t & \text{for} \quad 0 \le t \le 9 \\ 0.06 & \text{for} \quad 9 < t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time $t$. [5]

(ii) (a) Calculate the present value of £5,000 due at the end of 15 years.

(b) Calculate the constant force of interest implied by the transaction in part (a). [4]

A continuous payment stream is received at rate $100e^{-0.02t}$ units per annum between $t = 11$ and $t = 15$.

(iii) Calculate the present value of the payment stream. [4]

[Total 13]

 

**16** ***Subject CT1 April 2013 Question 5***

The force of interest per unit time at time $t$ , $\delta(t)$ , is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & \text{for } t < 6 \\ 0.07 & \text{for } t \geq 6 \end{cases}$$

(i) Calculate the total accumulation at time 10 of an investment of £100 made at time 0 and a further investment of £50 made at time 7. [4]

(ii) Calculate the present value at time 0 of a continuous payment stream at the rate $£50e^{0.05t}$ per unit time received between time 12 and time 15. [5]

[Total 9]

**17** ***Subject CT1 September 2013 Question 1***

The rate of interest is 4.5% per annum effective.

(i) Calculate:

(a) the annual effective rate of discount.

(b) the nominal rate of discount per annum convertible monthly.

(c) the nominal rate of interest per annum convertible quarterly.

(d) the effective rate of interest over a five year period. [5]

(ii) Explain why your answer to part (i)(b) is higher than your answer to part (i)(a). [2]

[Total 7]

**18** ***Subject CT1 September 2013 Question 10 (part)***

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t$$

Calculate the accumulated value of a unit sum of money:

(i) (a) accumulated from time $t = 0$ to time $t = 7$.

(b) accumulated from time $t = 0$ to time $t = 6$.

(c) accumulated from time $t = 6$ to time $t = 7$. [5]

(iv) Calculate the present value of an annuity that is paid continuously at a rate of $30e^{-0.01t+0.001t^2}$ units per annum from $t = 3$ to $t = 10$. [5]
[Total 10]

**19** ***Subject CT1 April 2014 Question 3***

£900 accumulates to £925 in four months.

Calculate the following:

(i) the nominal rate of interest per annum convertible half-yearly [2]

(ii) the nominal rate of discount per annum convertible quarterly [2]

(iii) the simple rate of interest per annum. [2]
[Total 6]

 

**20** ***Subject CT1 April 2014 Question 11***

An individual can obtain a force of interest per annum at time $t$ , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \le t < 4 \\ 0.07 & 4 \le t < 6 \\ 0.09 & 6 \le t \end{cases}$$

(i) Calculate the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of \$3,000 per annum from time $t = 4$ to $t = 10$ . [6]

(ii) Calculate the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]

[Total 9]

**21** ***Subject CT1 September 2014 Question 3***

A 91-day treasury bill is bought for £98.83 and is redeemed at £100.

(i) Calculate the annual effective rate of interest from the bill. [3]

(ii) Calculate the annual equivalent simple rate of interest. [2]

[Total 5]

## 22 *Subject CT1 September 2014 Question 5*

Calculate, at a rate of interest of 5% per annum effective:

(i) $a_{\overline{5}|}^{(12)}$ [1]

(ii) ${}_{4|}a_{\overline{15}|}$ [1]

(iii) $(I\bar{a})_{\overline{10}|}$ [1]

(iv) $(\bar{I}\bar{a})_{\overline{10}|}$ [1]

(v) the present value of an annuity that is paid annually in advance for 10 years with a payment of 12 in the first year, 11 in the second year and thereafter reducing by 1 each year. [2]

[Total 6]

## 23 *Subject CT1 September 2014 Question 7*

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for} \quad 0 < t \le 10 \\ 0.003t & \text{for} \quad 10 < t \le 20 \\ 0.0001t^2 & \text{for} \quad t > 20 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time $t = 28$. [7]

(ii) (a) Calculate the equivalent constant force of interest from $t = 0$ to $t = 28$.

(b) Calculate the equivalent annual effective rate of discount from $t = 0$ to $t = 28$. [3]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per unit time between $t = 3$ and $t = 7$.

(iii) Calculate the present value of the payment stream. [4]

[Total 14]

 

**24** ***Subject CT1 April 2015 Question 2***

Calculate the time in days for £3,000 to accumulate to £3,800 at:

(a) a simple rate of interest of 4% per annum.

(b) a compound rate of interest of 4% per annum effective. [4]

**25** ***Subject CT1 April 2015 Question 3***

A 182-day treasury bill, redeemable at $100, was purchased for $96.50 at the time of issue and later sold to another investor for $98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.

(i) Calculate the length of time in days for which the initial purchaser held the bill. [2]

(ii) Calculate the annual simple rate of return achieved by the second investor. [2]

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]

[Total 6]

**26** ***Subject CT1 April 2015 Question 5***

An investor pays £120 per annum into a savings account for 12 years. In the first four years, the payments are made annually in advance. In the second four years, the payments are made quarterly in advance. In the final four years, the payments are made monthly in advance.

The investor achieves a yield of 6% per annum convertible half-yearly on the investment.

Calculate the accumulated amount in the savings account at the end of 12 years. [7]

## 27 *Subject CT1 April 2015 Question 10*

The force of interest, $\delta(t)$, is a function of time and at any time $t$ (measured in years) is given by

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 4 \\ 0.12 - 0.01t & \text{for } 4 < t \le 9 \\ 0.05 & \text{for } t > 9 \end{cases}$$

(i) Determine the discount factor, $v(t)$, that applies at time $t$ for all $t \ge 0$. [5]

(ii) Calculate the present value at $t = 0$ of a payment stream, paid continuously from $t = 10$ to $t = 12$, under which the rate of payment at time $t$ is $100e^{0.03t}$. [4]

(iii) Calculate the present value of an annuity of £1,000 paid at the end of each year for the first three years. [3]

[Total 12]

## 28 *Subject CT1 September 2015 Question 1*

An investor wishes to obtain a rate of interest of 3% per annum effective from a 91-day treasury bill.

Calculate:

(a) the price that the investor must pay per £100 nominal.

(b) the annual simple rate of discount from the treasury bill. [3]

 

**29** ***Subject CT1 September 2015 Question 2***

The nominal rate of discount per annum convertible monthly is 5.5%.

(i) Calculate, giving all your answers as a percentage to three decimal places:

(a) the equivalent force of interest.

(b) the equivalent effective rate of interest per annum.

(c) the equivalent nominal rate of interest per annum convertible monthly. [3]

(ii) Explain why the nominal rate of interest per annum convertible monthly calculated in part (i)(c) is less than the equivalent annual effective rate of interest calculated in part (i)(b). [1]

(iii) Calculate, as a percentage to three decimal places, the effective annual rate of discount offered by an investment that pays £159 in eight years' time in return for £100 invested now. [1]

(iv) Calculate, as a percentage to three decimal places, the effective annual rate of interest from an investment that pays 12% interest at the end of each two-year period. [1]

[Total 6]

**30** ***Subject CT1 September 2015 Question 5***

An individual can obtain a force of interest per annum at time $t$, measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t \le 3 \\ 0.005 & t > 3 \end{cases}$$

(i) Determine the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of £5,000 per annum from time $t = 3$ to time $t = 6$. [5]

(ii) Determine the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]

(iii) Determine the amount an individual would accumulate from the investment of £300 from time $t = 0$ to time $t = 50$. [2]

[Total 10]

## 31 *Subject CT1 April 2016 Question 6*

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \le t \le 4 \\ 0.10 - 0.01t & 4 < t \le 7 \\ 0.01t - 0.04 & 7 < t \end{cases}$$

(i) Calculate, showing all working, the value at time $t = 5$ of £10,000 due for payment at time $t = 10$. [5]

(ii) Calculate the constant rate of discount per annum convertible monthly which leads to the same result as in part (i). [2]

[Total 7]

## 32 *Subject CT1 September 2016 Question 1*

The nominal rate of interest per annum convertible quarterly is 5%.

Calculate, giving all the answers as a percentage to three decimal places:

(i) the equivalent annual force of interest. [1]

(ii) the equivalent effective rate of interest per annum. [1]

(iii) the equivalent nominal rate of discount per annum convertible monthly. [2]

[Total 4]

**33** ***Subject CT1 September 2016 Question 2***

The nominal rate of interest per annum convertible quarterly is 2%.

Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

**34** ***September 2016 Question 3***

Describe the characteristics of a repayment loan (or repayment mortgage). [3]

**35** ***Subject CT1 September 2016 Question 6***

At the beginning of 2015 a 182-day commercial bill, redeemable at £100, was purchased for £96 at the time of issue and later sold to a second investor for £97.50. The initial purchaser obtained a simple rate of interest of 3.5% per annum before selling the bill.

(i) Calculate the annual simple rate of return which the initial purchaser would have received if they had held the bill to maturity. [2]

(ii) Calculate the length of time in days for which the initial purchaser held the bill. [2]

The second investor held the bill to maturity.

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]
[Total 6]

## 36 *Subject CT1 September 2016 Question 12*

The force of interest, $\delta(t)$, is a function of time and at any time $t$ (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for} \quad 0 \le t \le 10 \\ at & \text{for} \quad 10 < t \le 20 \\ bt & \text{for} \quad t > 20 \end{cases}$$

where $a$ and $b$ are constants.

The present value of £100 due at time 20 is 50.

(i) Calculate $a$. [5]

The present value of £100 due at time 28 is 40.

(ii) Calculate $b$. [4]

(iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between $t = 3$ and $t = 7$.

(iv) (a) Calculate, showing all workings, the present value of the payment stream.

(b) Determine the level continuous payment stream per annum from time $t = 3$ to time $t = 7$ that would provide the same present value as the answer in part (iv)(a) above. [8]

[Total 19]

**37** ***Subject CT1 April 2017 Question 1***

Calculate the nominal rate of discount per annum convertible monthly which is equivalent to:

(i) an effective rate of interest of 1% per quarter. [2]

(ii) a force of interest of 5% per annum. [2]

(iii) a nominal rate of discount of 4% per annum convertible every three months. [2]

[Total 6]

**38** ***Subject CT1 September 2017 Question 1***

(i) Calculate the time in days for £6,000 to accumulate to £7,600 at:

(a) a simple rate of interest of 3% per annum.

(b) a compound rate of interest of 3% per annum effective.

(c) a force of interest of 3% per annum. [6]

Note: You should assume there are 365 days in a year.

(ii) Calculate the effective rate of interest per half-year which is equivalent to a force of interest of 3% per annum. [1]

[Total 7]

**39** ***Subject CT1 September 2017 Question 3***

An investor is considering two investments. One is a 91-day deposit which pays a compound rate of interest of 3% per annum effective. The second is a government bill.

Calculate the annual simple rate of discount from the government bill if both investments are to provide the same effective rate of return. [3]

## 40 *Subject CT1 Septmber 2017 Question 6*

An investor has a choice of two 15-year savings plans, A and B, issued by a company. In both plans, the investor pays contributions of \$100 at the start of each month and the contributions accumulate at an effective rate of interest of 4% per annum before any allowance is made for expenses.

In plan A, the company charges for expenses by deducting 1% from the annual effective rate of return.

In plan B, the company charges for expenses by deducting \$15 from each of the first year's monthly contributions before they are invested. In addition it deducts 0.3% from the annual effective rate of return.

Calculate the percentage by which the accumulated amount in Plan B is greater than the accumulated amount in Plan A, at the end of the 15 years. [6]

## 41 *Subject CT1 Septmber 2017 Question 9*

The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \le t \le 10 \\ 0.06 & t > 10 \end{cases}$$

(i) Calculate the corresponding constant effective annual rate of interest for the period from $t = 0$ to $t = 10$. [4]

(ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible half-yearly. [1]

(iii) Calculate the accumulation at time $t = 15$ of £1,500 invested at time $t = 5$. [3]

(iv) Calculate the corresponding constant effective annual rate of discount for the period $t = 5$ to $t = 15$. [1]

(v) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time $t = 11$ to time $t = 15$. [6]

[Total 15]

 

# *SOLUTIONS TO PAST EXAM QUESTIONS*

*The solutions presented here are just outline solutions for you to use to check your answers. See ASET for full solutions.*

**1** ***Subject CT1 April 2008 Question 9***

(i) *Value at time 5 of £1,000 due at time 10*

The value at time 5 of £1,000 at time 10 is given by the expression:

$$\begin{aligned}
V_5 &= 1{,}000\exp\left(-\int_5^7 (0.1-0.01t)\,dt\right)\exp\left(-\int_7^{10} (0.01t-0.04)\,dt\right)\\
&= 1{,}000\exp\left(-\Big[0.1t-0.005t^2\Big]_5^7\right)\exp\left(-\Big[0.005t^2-0.04t\Big]_7^{10}\right)\\
&= 1{,}000\,\mathrm{e}^{-0.455+0.375}\,\mathrm{e}^{-0.1-0.035}\\
&= 1{,}000e^{-0.08}e^{-0.135}\\
&= 1{,}000e^{-0.215}\\
&= £806.54
\end{aligned}$$

(ii) *Equivalent rate of interest convertible monthly*

To find the value of $i^{(12)}$, we must solve the equation:

$$806.54\left(1+\frac{i^{(12)}}{12}\right)^{60} = 1{,}000 \quad\Rightarrow\quad i^{(12)} = 4.3077\%$$

(iii) *Accumulated value of payment stream*

The accumulated value of the payment stream at time 4 is:

$$V_4 = \int_0^4 100e^{0.02t} \exp\left(\int_t^4 0.06ds\right)dt = \int_0^4 100e^{0.02t} \exp\left(\left[0.06s\right]_t^4\right)dt$$

$$= \int_0^4 100e^{0.02t}e^{0.24-0.06t}dt$$

$$= 100e^{0.24}\int_0^4 e^{-0.04t}dt = 100e^{0.24}\left[-\frac{e^{-0.04t}}{0.04}\right]_0^4$$

$$= \frac{100e^{0.24}}{0.04}\left(1-e^{-0.04\times 4}\right) = 469.9052$$

The accumulated value of the payment stream at time 12 is:

$$V_{12} = 469.9052 \times A(4,7) \times A(7,12)$$

$$= 469.9052 \exp\left(\int_4^7 (0.1-0.01t)dt\right)\exp\left(\int_7^{12} (0.01t-0.04)dt\right)$$

$$= 469.9052 \exp\left(\left[0.1t-0.005t^2\right]_4^7\right)\exp\left(\left[0.005t^2-0.04t\right]_7^{12}\right)$$

$$= 469.9052\, e^{0.455-0.32}\, e^{0.24-(-0.035)}$$

$$= 469.9052 \times e^{0.135} \times e^{0.275}$$

$$= 708.0615$$

## 2 *Subject CT1 September 2008 Question 7*

(i) *Present value of £1,000 due at the end of 12 years*

The value at time 0 of a payment of £1,000 at time 12 is given by:

$$1{,}000 \exp\left(-\int_0^5 \left(0.05+0.02t\right) dt\right)\exp\left(-\int_5^{12} 0.15\ dt\right)$$

 

Now:

$$\exp\left(-\int_0^5 (0.05+0.02t)\,dt\right) = \exp\left(-\left[0.05t+0.01t^2\right]_0^5\right)$$
$$= \exp(-0.05\times 5-0.01\times 25) = \exp(-0.5)$$

and:

$$\exp\left(-\int_5^{12} 0.15\ dt\right) = \exp\left(-\left[0.15t\right]_5^{12}\right)$$
$$= \exp(-0.15\times 12+0.15\times 5) = \exp(-1.05)$$

Therefore, the value at time 0 is:

$$1{,}000e^{-0.5}e^{-1.05} = 1{,}000e^{-1.55}$$
$$= £212.25$$

(ii) *Equivalent rate of discount*

Let $d$ be the equivalent annual rate of discount. Then:

$$1{,}000(1-d)^{12} = 1{,}000e^{-1.55}$$
$$\Rightarrow \quad (1-d)^{12} = e^{-1.55}$$

Hence:

$$d = 1-\left(e^{-1.55}\right)^{1/12} = 12.12\%$$